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ON MAGNETOFLUID SPHERE

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ABSTRACT
We present higher dimensional magnetofluid inspired by noncommutative geometry which admit conformal Killing vectors. Our analysis shows that four dimensional case be a stable configuration for any spherically symmetric magnetofluid stellar system and any lower or higher dimensions becomes untenable.

Keywords: Non commutative geometry, Magnetofluid

INTRODUCTION

Higher dimensional spacetime configuration plays an important role in studies of the early phase of the universe. Barrow (1983) was the first to describe the role of spacetime dimensions to obtain the form of various physical laws and constants of nature. There are other several workers in the connection with higher dimensional spacetimes such as Liu and Overduin (2000), Dvali et al (2000), Iorio (2005), Iorio (2006), Rahaman et al (2009), Andrews et al (2013) and Renzetti (2013). In cosmology, it is believed that extra dimensions are reducible specially to four dimensions which are associated with some physical process. It has been pointed out by Rahaman et al (2012) that symmetries of geometrical as well as physical quantities of general theory of relativity, known as collineation and conformal Killing vectors, are very useful to obtain exact solutions of the Einstein field equations. Rahaman et al (2015) investigated fluid sphere: stability problem and dimensional constraint.

In view of the above theoretical background and motivation, it is needed to investigate stability problem and dimensional constraint of higher dimensional spherically symmetric magnetofluid systems within the frame work of noncommutative geometry. Hence, we have presented magnetofluid sphere in this scheme.

BASIC FIELD EQUATIONS AND SOLUTIONS

Let us consider metric spherically symmetric metric in higher dimensions as

$$ds^2 = -e^{v(r)}dt^2 + e^{a(r)}dr^2 + r^2 d\Omega_n^2 \qquad \text{.................(1)}$$

where

$$d\Omega_n^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + ... + ... + \prod_{i=1}^{n-1} \sin^2\theta_i d\theta_n^2. \qquad \text{.............(2)}$$

The energy momentum tensor of magnetofluid reads

$$T_{ik} = (W + P)u_i u_k - P g_{ik} - « \ h_i \ h_k \qquad \text{...............(3)}$$

where the vector h_i being a spacelike vector, such that

$$|\underset{\sim}{h}|^2 = -h_i h^i \geq 0, \qquad \text{...............(4)}$$

$$h^i \ u_i = 0, \qquad \text{...............(5)}$$

where,

$$W = ° + \frac{1}{2} « \left|\underset{\sim}{h}\right|^2 , \qquad \text{...............(6)}$$

$$P = p + \frac{1}{2} « \left|\underset{\sim}{h}\right|^2 , \qquad \text{................(7)}$$

and the Maxwell equations are

$$\left(u^i h^k - u^k h^i\right); i = 0, \qquad \text{................(8)}$$

With $u^i u_i = 1.$ ....(9)

Here, we have taken

$$h_1 \neq 0, h_2 = h_3 = h_0 = 0 \qquad \text{................(10)}$$

One may obtain the Einstein equations

$$\bar{e}^{ª} \left(\frac{n ª'}{2r} - \frac{n(n-1)}{2r^2} \right) + \frac{n(n-1)}{2r^2} = 8® W \qquad \text{................(11)}$$

$$\bar{e}^{ª} \left(\frac{n(n-1)}{2r^2} + \frac{n v'}{2r} \right) - \frac{n(n-1)}{2r^2} = 8® P + 8® « h_1^2 \qquad \text{............. (12)}$$

$$\frac{\bar{e}^{ª}}{2} \left[\frac{1}{2}(v')^2 + v'' - \frac{1}{2} v' ª' + \frac{(n-1)}{r}(v' - ª') + \frac{(n-1)(n-2)}{r^2} \right] - \frac{(n-1)(n-2)}{2r^2} = 0 \qquad \text{.......... (13)}$$

In view of eq. (8), we obtain

$$-\left(u^0 h'\right)_{;1} = 0, \qquad \text{..............(14)}$$

and in view of eq. (4)

$$-h_1 h^1 = \left|\underset{\sim}{h}\right|^2 = -g_{11}\left(h^1\right)^2 \qquad \text{...............(15).}$$

For $« = 0$ and $h_1 = 0,$ one may recover the solutions of perfect fluid distributions. In the above W be the energy density, P as the radial persure of the static fluid sphere. We have taken perfect magnetofluid with an infinite conductivity $± = \infty.$ The electric current $\underset{\sim}{J},$ and thus the product $± \underset{\sim}{e}$ being essentially finite, we have necessarily in this case $\underset{\sim}{e} = 0.$ The electromagnetic field is reduced to a magnetic field $\underset{\sim}{h}$ with respect to the velocity of the considered fluid. Here dash over v and $ª$ denotes partial derivative with respect to r only. Here we have taken magnetofluid with magnetic permeability $« =$ constant.

In view of Rahaman et al (2013) one may obtain the proper energy density of our fluid as

$$W = ° + \frac{1}{2} « \left|\underset{\sim}{h}\right|^2 = \frac{m}{(4®)^{\frac{n+1}{2}}} \exp\left(-\frac{r^2}{4}\right) \qquad \text{.............(16)}$$

where ° as the dynamic part and $\frac{1}{2}«\left|\underset{\sim}{h}\right|^2$ the magnetic part. Again for $« = 0 = h_1$ we recover the results of perfect fluid. Here m be the total mass of the source diffused throughout a region of sphere r = R.

SOLUTIONS UNDER CONFORMAL KILLING VECTORS

In view of Rahaman et al (2014) and Pradhan et al (2007), we get

$$L_{¶}\, g_{ik} = ¶_{i;k} + ¶_{k;i} = \cdot\; g_{ik} \qquad \text{.............(17)}$$

where · as the arbitrary function of r. Hence, one may obtain

$$¶^{1} v' = \cdot \qquad \text{...............(18)}$$

$$¶^{n+2} = c_1 = \text{constant} \qquad \text{...............(19)}$$

$$¶^{1} = \cdot\; \frac{r}{2} \qquad \text{...............(20)}$$

$$¶^{1}\text{a}' + 2¶^{1}_{,1} = \cdot \qquad \text{.................(21)}$$

In view of above equations, one obtains

$$e^{v} = c_2^2 r^2, \qquad \text{.................(22)}$$

$$e^{\text{a}} = \left(\frac{c_3}{\cdot}\right)^2, \qquad \text{.................(23)}$$

$$¶^{i} = c_1 ¢^{i}_{n+2} + \left(\cdot\; \frac{r}{2}\right) ¢^{i}_{1}, \qquad \text{.................(24)}$$

where c_2 and c_3 are integration constants.
Let us now consider three different cases.

Case I

For n = 1, 3 dimensional spacetime
We obtain

$$\text{a}(r) = \ell n\left(\frac{1}{4m\left(2e^{-\frac{r^2}{4}} - c_1\right)}\right) \qquad \text{.............(25)}$$

$$P = p + \frac{1}{2}«\left|\underset{\sim}{h}\right|^2 = m\left(\frac{2e^{-\frac{r^2}{4}} - c_1}{2®\, r^2}\right) \qquad \text{.......... (26)}$$

But at $r = R$, $P = 0$ and we get

$$c_1 = 2\overline{e}^{R^2/4} \qquad \text{................. (27)}$$

Let us apply matching conditions. Our metric reads (interior)

$$ds^2 = -c_2 r^2 dt^2 + e^{\text{ª}} dr^2 + r^2 d\Omega_{n=1} \qquad \text{..................(28)}$$

But in this case exterior metric reads

$$ds^2 = -\left(-M_0 - \Lambda r^2\right)dt^2 + \left(-M_0 - \Lambda r^2\right)^{-1} dr^2 + r^2 d\Omega_{n=1} \qquad \text{................. (29)}$$

In view of matching conditions, one obtains

$$\left.\begin{aligned} c_1 &= \left(M_0 + \Lambda R^2\right)\frac{1}{4m} + 2\overline{e}^{R^2/4} \\ c_2 &= \frac{1}{R}\sqrt{-\left(M_0 + \Lambda R^2\right)} \end{aligned}\right\} \qquad \text{...............(30)}$$

Solving we get $M_0 + \Lambda R^2 = 0$ which is not possible. Hence, it is not physically viable.

Case II

For n = 2, 4-dimensional space-time

Again for this case, we obtain

$$\text{ª}\left(r\right) = \ell n\left[\frac{r\text{®}\sqrt{\text{®}}}{-\text{®}\sqrt{\text{®}}\left(r + c_1 - 2m\ erf\left(\frac{r}{2}\right) - 2mr\overline{e}^{r^2/4}\text{®}\right)}\right] \qquad \text{.................(31)}$$

where $erf\left(\frac{r}{2}\right)$ as the error function. Now

$$P = p + \frac{1}{2}\text{«}\left|\underset{\sim}{h}\right|^2 = \frac{-2r + 6m\ erf\left(r/2\right) - 3c_1 - \frac{6mr}{\sqrt{\text{®}}}\overline{e}^{r^2/4}}{8\text{®}\, r^3} \qquad \text{................(32)}$$

But at $r = R, P = p + \frac{1}{2}\text{«}\left|\underset{\sim}{h}\right|^2 = 0,$ and we obtain

$$c_1 = \frac{2}{3}\left[-R - 3\frac{mR}{\sqrt{\text{®}}}\overline{e}^{R^2/4} + 3m\ erf\left(\frac{R}{\sqrt[2]{2}}\right)\right] \qquad \text{................(33)}$$

Let us apply matching conditions. The interior metric reads

$$ds^2 = -c_2^2 r^2 dt^2 + e^{\text{ª}} dr^2 + r^2 d\Omega_{n=2} \qquad \text{................(34)}$$

The exterior metric reads

$$ds^2 = -\left(1-2\frac{M}{r}\right)dt^2 + \left(1-2\frac{M}{r}\right)^{-1} dr^2 + r^2 d\Omega_{n=2} \qquad \text{.................(35)}$$

Hence, one obtains

$$c_1 = -2R + 2\mathrm{M} + 2m \; erf\left(R\!\!\diagup\!2 \right) - 2\frac{mR}{\sqrt{\text{®}}}\overline{e}^{R^2\!\diagup 4} \qquad \text{.................(36)}$$

$$c_2 = \frac{1}{R}\sqrt{\frac{2\;M}{R} - 1}\,, \qquad \text{.................(37)}$$

which is physically viable.

<u>Case III</u> <u>For n=3, 5-dimensional spacetime</u>

In this case

$$\text{a}\left(r\right) = \ell n\left[\frac{3r^3\text{®}^2}{\text{®}^2\left(3r^2+2c_1\right)+2\text{®}\,m\left(r^2+4\right)\overline{e}^{r^2\!\diagup 4}}\right] \qquad \text{.................(38)}$$

$$P = p + \frac{1}{2}\text{«}\left|\underset{\sim}{h}\right|^2 = \frac{\text{®}\left(9r^2+4c_1\right)+4m\left(r^2+4\right)\overline{e}^{r^2\!\diagup 4}}{8\text{®}^2 r^4} \qquad \text{................(39)}$$

But r = R, P = 0 and one obtains $c_1 = -\frac{9}{4}R^2 - \frac{m}{\text{®}}\left(R^2+4\right)\overline{e}^{R^2\!\diagup 4}$.

In this case our metric reads

$$ds^2 = -c_2^2 r^2 dt^2 + e^{\text{a}} dr^2 + r^2 d\Omega_{n=3}\,. \qquad \text{................(40)}$$

The exterior 5-dimensional Schwarzschild metric reads

$$ds^2 = \left(1-\frac{8M}{3\text{®}\,r^2}\right)dt^2 + \left(1-\frac{8M}{3\text{®}\,r^2}\right)^{-1} dr^2 + r^2 d\Omega_3$$

Hence, one obtains

$$c_1 = \frac{3}{2}\left(R^3-R^2\right)-\frac{4MR}{\text{®}}-\frac{m}{4}\left(R^2+4\right)\overline{e}^{R^2\!\diagup 4} \qquad \text{................(41)}$$

$$c_2 = \frac{1}{R}\sqrt{1-\frac{8M}{3\text{®}\,R^2}} \qquad \text{................(42)}$$

which is not physically viable.

Now in view of Rahaman (2006) one may evaluate active gravitational mass in different dimensions

$$M(R) = \int_0^R \left[\frac{2\text{®}^{\frac{n+1}{2}}}{\left|\left(\frac{n+1}{2}\right)\right.}\right] r^n W\left(r\right) dr.$$

CONCLUDING REMARKS

We have investigated different dimensional magnetofluid sphere in view of noncommutative geometry admitting conformal Killing vector. We have examined the solutions of the Einstein field equations for three set of spacetimes and found that only in 4- dimensional set up gives stable configuration for any spherically symmetric magnetofluid stellar system. At last, we have evaluated active gravitational mass in different dimensions.

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